

Regular Rings and Separativity Problem for PI, PM, GPI, Self-Pseudo Injective, and C * Algebras

Abstract

Separative von Neumann regular rings exist in abundance. For example, all regular self-injective rings, unit regular rings, regular rings with a polynomial identity are separative. It remains open whether there exists a non-separative regular ring. We give a variety of conditions under which a von Neumann regular ring is separative. Jain- Leroy showed that a von Neumann regular ring R is separative under anyone of the following cases: (1) R is CS; (2) R is pseudo injective; (3) R satisfies the closure extension property: the essential closures in R of two isomorphic right ideals are themselves isomorphic. Note that all PI- algebras, Pivotal Monomial Algebras, Generalized PI algebras also imply that regular rings having any one of these properties are separative. Another test to check for separativity is given in (Jain- Leroy, J. Optimization Theory, 2024) which states that if a block matrix $\begin{pmatrix} B & 0 \\ 0 & 0 \end{pmatrix}$ over a regular ring cannot be expressed as a product of idempotents, then that regular ring is not separative.